Exp:- set $A = \{a, b, c\}$, $R = \{(a,a)(b,b), (c,c), (a,b)(b,a)$, R

$[a] = \{a, b\}$

$[b] = \{b, a\} = [a]$ $[a]$ & $[b]$ are same.

$[c] - \{c\}$

Two different classes so Rank of R is 2.

Exp: $A = \{1, 2, 3, 4\}$,

$R = \{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)(4,4)\}$

$[1] = \{1, 2, 3\}$

$[2] = \{1, 2, 3\}$

$[3] = \{1, 2, 3\}$

$[4] = \{4\}$

So Rank is 2. because $[1]$, $[2]$ and $[3]$ are same.

Order of Relation:-

(i) **Partial oreder Relation:** (a) Reflexive

(b) Antisymmetric

(c) Transitive

Imp

* (A, R) is Poset.

Exp: set of Integer relation $\leq$.

ii) **Total order or Linear order:** consider the relⁿ

(a) ~~Partial order~~ R on a set A: If it is the case

(b) for all $a, b \in A$ we have either $(a, b) \in R$ or $(b, a) \in R$ or $a = b$ then R is called total order Relation.

Exp: $<$ for integer

(iii) **Quasi order:** (a) Irreflexive (b) Transitive

Exp: Proper subset of power set of a set

Representation of Relation:

(i) Graphical and Tabular form:

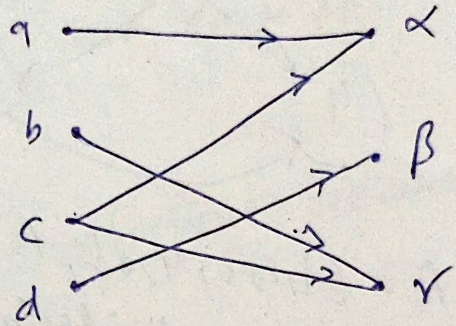
Let $A = \{a, b, c, d\}$, $B = \{\alpha, \beta, \gamma\}$

$R = \{(a,\alpha), (b,\gamma), (c,\alpha), (c,\gamma), (d,\beta)\}$

Tabular form

	α	β	γ
a	✓		
b			✓
c	✓		✓
d		✓	

Graphical form



(ii) Matrix form

Represent the matrix M of order $m \times n$ where m & n are number of elements in A & B set and R is a relation from $A \to B$:

then $M_{ij} = 1$ if $(a_i, b_j) \in R$

$M_{ij} = 0$ otherwise.

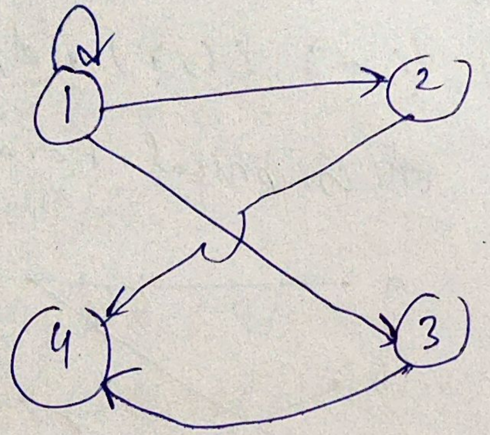
Exp: $A = \{a, b, c, d\}$, $B = \{\alpha, \beta, \gamma\}$

$R = \{(a,\alpha), (b,\gamma), (c,\alpha), (c,\gamma), (d,\beta)\}$

$$\begin{array}{c c} & \begin{array}{ccc} \alpha & \beta & \gamma \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

(iii) Directed Graph or digraph :- if R is on a set A. $A = \{1, 2, 3, 4\}$

$R = \{(1,1)\ (1,2)\ (1,3)\ (2,4)\ (4,3)\}$



Exp:- $A = \{1,2,3,4,8\}$, $B = \{1,4,6,9\}$. Find Relation matrix.
Let aRb iff a divides b.

Exp:- $A = \{1,2,3,4,8\} = B$.
if aRb iff $a+b \le 9$ Find Relation & represt by matrix.

Circular Relation: A relation R is said to be circular if $(x,y) \in R$ and $(y,z) \in R \Rightarrow (z,x) \in R$.

Operation of a Relation:

(i) Inverse of a relation.
(ii) Intersection and union of a Relation
(iii) complement of a Relation

complement of a Relation: consider a relation R from set A to B. The complement of Relation R denoted by $\overline{R}$ is a relation from A to B such That $\overline{R} = \{(a,b) : (a,b) \in U \text{ and } (a,b) \notin R\}$

Type of Relation & Properties (3) of Relation :-

(1) Reflexive Relation:

A relation R on a set A is a reflexive if aRa for every $a \in A$ i.e. if $(a,a) \in R$ for every $a \in A$.

Exp:- (1) Relation $\parallel$ (Parallel) on the set of lines in the plane is Reflexive because each line is parallel to itself.

Exp:- (2) Relation '$x = y$' is reflexive because $x = x$ for every x.

Exp:- (3) In real number $x < y$ is not reflexive because $x \not< x$.

(2) Symmetric Relation:

A relation R on a set A is symmetric if whenever aRb then bRa i.e. whenever $(a,b) \in R$ then $(b,a) \in R$.

Exp:- Let A be the set of all straight line in a plane the relation R is defined by "a is perpendicular to b".

Ans:- The relation is symmetric beause $a \perp b$ then $b \perp a$.

Exp:-② set of natural number, relation R in N is defined by "a is equal to b".

ans:- R is symmetric because $a=b \Rightarrow b=a$

$aRb \Rightarrow bRa$

③ **Transitive Relation:** A relation R on a set A is transitive if whenever aRb and bRc then aRc i.e. if whenever $(a,b), (b,c) \in R$ then $(a,c) \in R$.

Exp:- which of the relation are transitive

$R_1 = \{(1,1)\ (1,2)\ (2,3)\ (1,3)\ (4,4)\}$

$R_2 = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,3)\ (4,4)\}$

$R_3 = \{(1,3)\ (2,1)\}$

$R_4 = \phi$ the empty relation

$R_5 = A \times A$ the universal Relation

ans:- R_3 is not transitive because $(2,1), (1,3) \in R$ but $(2,3) \notin R$.

Equivalence Relation:- Any Relation have given three properties then it is equivalence relation

(i) Reflexive aRa

(ii) Symmetric $aRb \Rightarrow bRa$

(iii) Transitive aRb and $bRc \Rightarrow aRc$

Closure of Relation: Let R be a relation on a set A. R may or may not have some property P, such as reflexivity, symmetry, transitivity. If there is a relation S with property P containing R such that S is a subset of every relation with P containing R, then S is called the closure of R with respect to P.

(i) Reflexive closure: Let R be a relation ($\subseteq A \times A$) on set A and R is not reflexive. Let S be a relation ($\subseteq A \times A$) on A which

(a) is reflexive (b) contain R (i.e. $R \subseteq S$)

(c) is a subset of every reflexive relⁿ on A that contain R.

Then S is called reflexive closure of R with respect to reflexivity.

$$\boxed{R^R = R \cup \Delta}$$

where $\Delta = \{(a,a) : a \in A\}$

(ii) Symmetric closure: Let R be a relation ($\subseteq A \times A$) on a set A and R is not symmetric. Let S be a relation ($\subseteq A \times A$) on which is

(a) is symmetric (b) contain R (i.e. $R \subseteq S$)

(c) is a subset of every symmetric relation on A ~~with~~ that contain R.

Then S is called symmetric closure of R with respect to symmetry.

$\boxed{R^S = R \cup R^{-1}}$ where R^{-1} is inverse relation of R.

8) **Transitive closure:** Let R be a relation on a set A and R is not transitive. Let S be a relation ($\subseteq A \times A$) on A which

(a) is transitive (b) contain R ($R \subseteq S$)

(c) is subset of every transitive relation on A that contain R.

Then S is called transitive closure of R with respect to transitivity.

$$\boxed{R^T = R \cup R^2 \cup R^3 \cup \cdots R^n}$$

where n is number of elements in set A.

and $R^2 = R \cdot R$

$R^3 = R^2 \cdot R$

$\vdots$

$R^n = R^{n-1} \cdot R$

Ex Q) Let A denote the set of real numbers and $B = A \times A$. A relation R is defined on A such that $(a,b) R (c,d)$ iff $a^2+b^2 = c^2+d$. Show that R is an equivalence relation.

Ans:- (i) $(a,b) R (a,b) \Leftrightarrow a^2+b^2 = a^2+b^2$

So R is reflexive

(ii) $(a,b) R (c,d) \Rightarrow a^2+b^2 = c^2+d^2$

$\Rightarrow c^2+d^2 = a^2+b^2$

$\Rightarrow (c,d) R (a,b)$

R is symmetric

(iii) $(a,b) R (c,d)$ and $(c,d) R (e,f)$

$\Rightarrow a^2+b^2 = c^2+d^2$ and $c^2+d^2 = e^2+f^2$

$\Rightarrow a^2+b^2 = e^2+f^2$

$\Rightarrow (a,b) R (e,f)$

So R is transitive

So R is an equivalence relation.

Q) Let $N = \{1,2,3 - - -\}$ and a Relation is defined in $N \times N$ as follows: (a,b) is related to (c,d) iff $ad = bc$, then show whether R is equivalence relation or not.

Ans: $(a,b) R (c,d) \Rightarrow ad = bc$

(i) $(a,b) R (a,b) \Rightarrow ab = ba$

we ~~note~~ know that for all natural number it is true so it is reflexive.

(ii) $(a,b) R (c,d) \Rightarrow ad = bc$

$\Rightarrow bc = ad$

$\Rightarrow cb = da$

$\Rightarrow (c,d) R (a,b)$

so symmetric

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(iii) If $(a,b) R (c,d)$ and $(c,d) R (e,f)$

$\Rightarrow ad = bc$ and $c.f = de$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$

$\Rightarrow \frac{a}{b} = \frac{e}{f}$

$\Rightarrow af = be$

$\Rightarrow (a,b) R (e,f)$

So R is transitive.

So R is equivalence relation

$(a,b) R (c,d) \Rightarrow a+d = b+c$. similar

Q) Let I is set of Integers and R =

$\{(x,y) \mid x-y$ is divisible by 3 $\}$ show that R is equivalence relation.

Ans:- (i) $xRx = x-x=0$ which is divisible by 3.

It is true for all integer.

so R is reflexive.

(ii) $xRy \Rightarrow x-y$ is divisible by 3

$yRx = y - x$

$= -(x-y)$

$\because x-y$ is divisible by 3.

$\therefore -(x-y)$ is also divisibly by 3.

so $xRy \Rightarrow yRx$.

so it is ~~reflexive~~ symmetric.

(iii) xRy and $yRz \Rightarrow x-y$ and $y-z$ divisibly 3.

$xRz = x - z$

$= \underline{x-y} + \underline{y-z}$

$\because x-y$ is divisible by 3 and $y-z$ is divisible by 3. so $x-y+y-z$ is also divisible by 3.

$\therefore xRy$ and yRz then xRz.

so it transitive.

Hence R is equivalence Relation. 10

Q) N is set of natural number and R is defind as $aRb = 3a+4b$ is divide by 7. is equivalence relation on nat

(i) $aRa = 3a + 4a$
$= 7a$
which is divisible by 7.
so it is reflexive.

(ii) if $aRb = 3a + 4b$ is divisible by 7.

$bRa = 3b + 4a$
$= 3b + 4a + 3a + 4b - 3a - 4b$
$= 7a + 7b - (3a + 4b)$
$= \underline{7(a+b)}$ ~~(3(a+b)~~ $- \underline{(3a + 4b)}$

$7(a+b)$ is divisible by 7 and $(3a+4b)$ is also divisible by 7.
$\Rightarrow aRb = bRa$ so it is symmetric.

(iii) if aRb and bRC then $3a+4b$ and $3b+4C$ ~~are~~ divisible by 7.

$aRC = 3a + 4C$
$= 3a + 4b + 4C + 3b - 7b$
$= \underline{3a + 4b} + \underline{3b + 4C} - 7b$

$3a+4b$, $3b+4C$, and $7b$ are divisible by 7. So $3a+4b+3b+4C-7b$ is divisible by 7.
Hence aRb and bRC Then aRc.
so transitive. Hence equivalence relation

Exp:- Relation "=" is equivalence relation or not. (3)

(i) Every integer is equal to itself so $a = a$ $(a, a) \in R$ for every $a \in A$.

so it is reflexive.

(ii) for integer if $a = b$ then $b = a$

$\therefore$ aRb then bRa.

So it is symmetric.

(iii) for integer if $a = b$ and $b = c$ then

$a = c$.

So it is transitive.

So it has three properties so this relⁿ is equivalence relation.

Antisymmetric Relation:- a Relation R in in a set S is antisymmetric if whenever aRb and bRa then $a = b$.

Composite Relation: Let R_1 be a relation from A to B and R_2 be a relation from B to C. Then the composition of relation R_1 and R_2 is denoted by $R_1 \circ R_2$ (or $R_2 \circ R_1$) and is defined as

$R_1 \cdot R_2 = \{(a,c) \mid a \in A \wedge c \in C \wedge \exists[b \in B \wedge (a,b) \in R_1 \wedge (b,c) \in R_2]\}$

Exp: ① Let $A = \{2, 3, 4, 5, 6\}$ and Let R_1 and R_2 be the relation on A such that

$R_1 = \{(a, b) \mid a - b = 2\}$

$R_2 = \{(a, b) \mid a + 1 = b \text{ or } a = 2b\}$ → find $R_1 \cdot R_2$

Ans: $R_1 = \{(4, 2), (5, 3), (6, 4)\}$

$R_2 = \{(2, 3), (3, 4), (4, 5), (5, 6), (4, 2), (6, 3)\}$

$R_1 \cdot R_2 = \{(4, 3)(5, 4), (6, 5), (6, 2)\}$

Exp: ② Let $x = \{4, 5, 6\}$, $y = \{a, b, c\}$, $z = \{l, m, n\}$

R_1 from x to y and $R_1 = \{(4, a)(4, b)(5, c)(6, a)(6, c)\}$

R_2 from y to z and $R_2 = \{(a, l)(a, n)(b, l)(b, m)(c, l)(c, m)(c, n)\}$

Find (i) $R_1 \cdot R_2$ and (ii) $R_1 \cdot R_1^{-1}$

(i) $R_1 \cdot R_2 = \{(4, l)(4, n)(4, l)(4, m)(5, l)(5, m)(5, n)(6, l)$
$(6, n), (6, m)\}$

(ii) ~~Universal Relation of R_1~~
~~$\{(4, a)(4, b)(4, c)(5, a)(5, b)(5, c)(6, a)(6, b)(6, c)\}$~~

~~$R^{-1} = \{(4, 4)(5, 4)(5, 6), (6, 6)\}$~~

$= \{(a, 4)(b, 4)(c, 5)(a, 6)(c, 6)\}$

$\therefore$ ~~$R_1 \cdot R_1^{-1}$~~ $=$ $R_1^{-1} = \{(a, 4)(b, 4)(c, 5)(a, 6)(c, 6)\}$

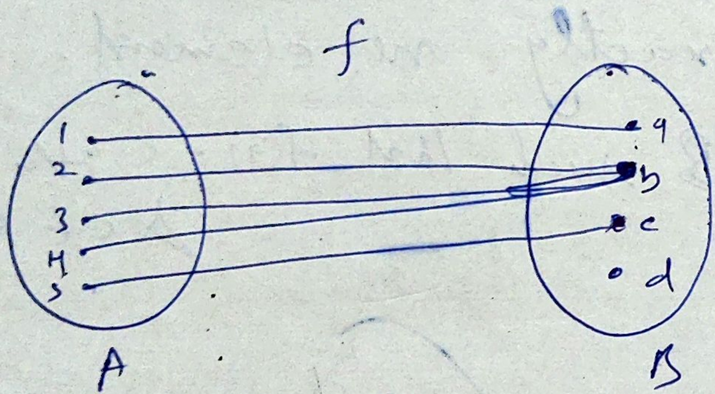
$R_1 \cdot R^{-1} = \{(4, 4)(4, 6)(5, 5)(5, 6)(6, 4)(6, 6)(6, 5)\}$

Unit-1

function: Let A & B are two non-empty sets. A rule which assign to each element $x \in A$ to a unique element $y \in B$ is called a function or mapping. written $f : A \to B$

The element $y \in B$ is called the image of x under f.

The element $x \in A$ is called pre-image of y under f.



Domain = set $A = \{1, 2, 3, 4, 5\}$

Co-Domain = set $B = \{a, b, c, d\}$

Range = (a, b, c)

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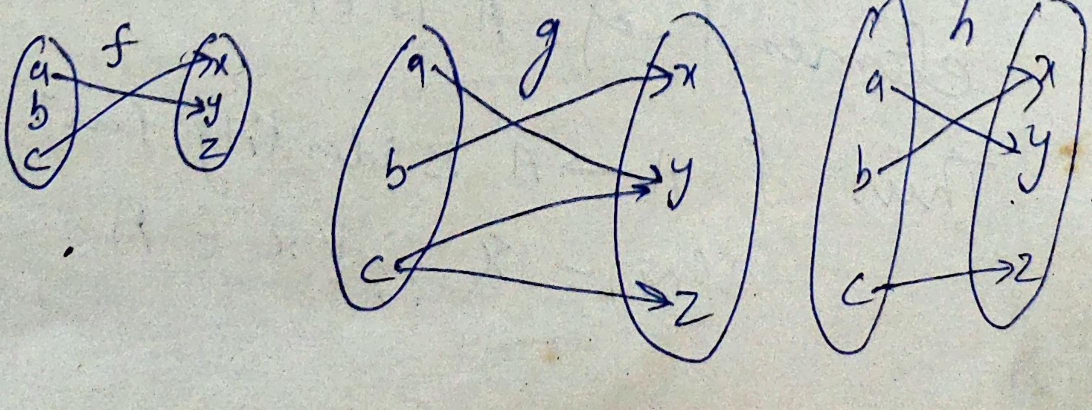
A – condition for function

① every element $a \in A$ has an image in B.

② an element $a \in A$ has only one image in B.

Exp: ①



$f: X \rightarrow X$

② $A = \{1, 2, 3, 4\}$ whether a fun from X into X

(a) $f = \{(2,3)\ (1,4)\ (2,1)\ (3,2)\ (4,4)\}$.

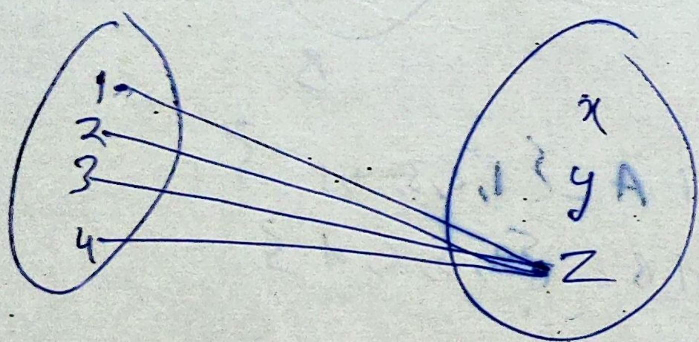
(b) $g = \{(3,1)\ (4,2)\ (1,3)\}$

(c) $h = \{(2,1)\ (3,4)\ (1,4)\ (2,1)\ (4,4)\}$

Constant function:

range exactly one element.

$f: A \rightarrow B$ such that $f(a) = c;\ \forall a \in A$ & $c \in B$



IDentity function:—

$f: A \rightarrow A$ is said to be identity function if f associates every element of A to element itself.

Thus $f: A \rightarrow A$ is identity fun^n iff

$f(x) = x\ ;\ \forall x \in A$.

Equal function:

Two function g & f are equal iff

(1) the domain of g = domain of f ;

(2) Co- ________ = Co- ________

(3) $f(x) = g(x)$ for every x belonging to their common domain

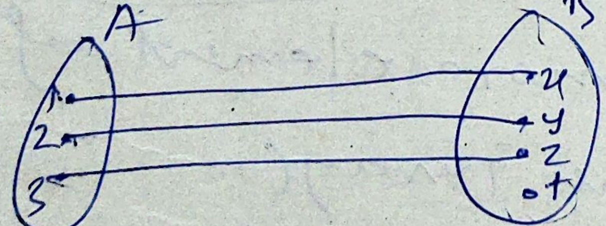
Exp:- $A = \{1, 2\}$, $B = \{3, 6\}$

$f : A \to B$; $f(x) = x^2 + 2$

$g : A \to B$; $g(x) = 3x$

Kind of function:

Into :- (Injective)

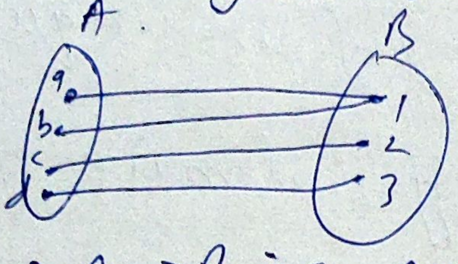


(21)

$f : A \to B$

If $f : A \to B$ is such that there is at least one element $b \in B$ that has no pre-image into set A then fun is called into.

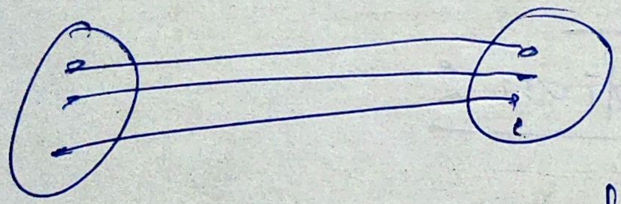
② Onto mapping (Surjective mapping):



If $f : A \to B$ is such that each element $b \in B$ have atleast one pre image into set A then it is called onto.

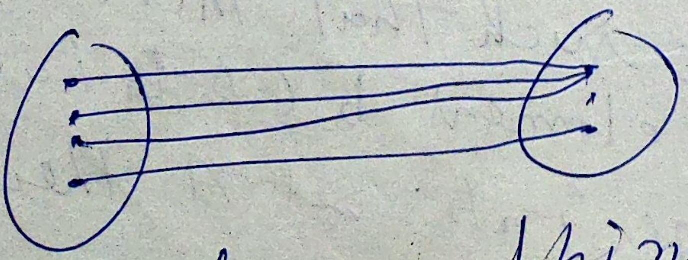
③ One-One mapping:

If different element of A have different image.

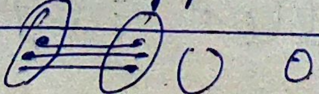


④ many-one mapping:-

If two or more element of A have same image.

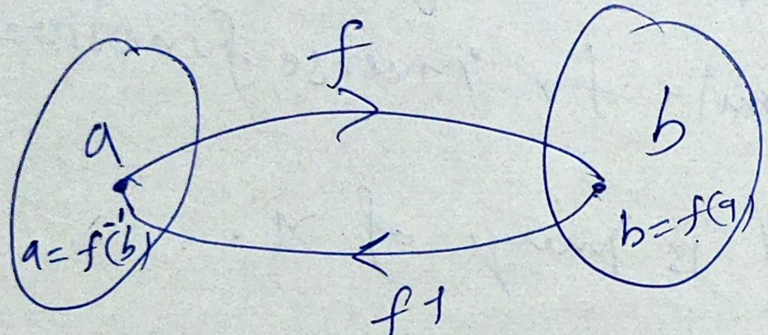


⑤ Bijection mapping: one-one & onto



Inverse of a function : (3) CAN_ FLASH

if $f : A \to B$ be a one-one onto mapping, III.
then the mapping $f^{-1} : B \to A$ which associate to each element $b \in B$ the unique element $a \in A$, is called the inverse of the map $f : A \to B$



Exp:- (1) Let $f : R \to R$ be defined by $f(x) = 2x - 3$. Now if f is one-one & onto, hence f has an inverse function f^{-1}. find formula for f^{-1}

ans:- Let y be the image of x under f

$$y = f(x) =$$

(22)

$$y = 2x - 3$$

$$x = \frac{y+3}{2}$$

Now x will be the image of y

$f^{-1}(y) = \frac{y+3}{2}$

put $y = x$

$\boxed{f^{-1}(x) = \frac{x+3}{2}}$

Q: Let $f: R \to R$ by $f(x) = 3x - 7$. Find formula for inverse function.

Ans. Let y is image of x.

$y = f(x)$

$y = 3x - 7$

$x = \frac{y+7}{3}$

So x is image of y under f^{-1}

$\boxed{f^{-1}(y) = \frac{y+7}{3}}$

put $y = x$

$\boxed{f^{-1} x = \frac{x+7}{3}}$

Operation of function:

① **Addition of two function:**

Let $f(x)$ and $g(x)$ be two functions, then their addition will be a function $(f+g)(x)$ is defined as

$$\boxed{(f+g)(x) = f(x) + g(x)}$$

② **Subtraction of two function:**

Let $f(x)$ and $g(x)$ be two function then their subtraction is defines as

$$\boxed{(f-g)(x) = f(x) - g(x)}$$

③ **Multiplication of two functions:**

Let $f(x)$ and $g(x)$ be two function then their multiplication is defined as

$$\boxed{(f \times g)(x) \text{ or } (f \cdot g)(x) = [f(x)] \cdot [g(x)]}$$

④ **Division of two function:**

Let $f(x)$ and $g(x)$ be two function, then their division is defined as

$$\boxed{(f/g)(x) = \frac{f(x)}{g(x)}}$$

Exp:-

$f(x) = 3x+2$

$g(x) = 4-5x$

find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, $(f/g)(x)$

(i) $(f+g)(x) = f(x) + g(x)$

$= 3x+2+4-5x$

$= 6-2x$

(ii) $(f-g)(x) = f(x) - g(x)$

$= (3x+2) - (4-5x)$

$= 3x+2-4+5x$

$= 8x-2$

(iii) $(f\cdot g)(x) = [f(x)]\cdot[g(x)]$

$= (3x+2)(4-5x)$

$= 12x - 15x^2 + 8 - 10x$

$= -15x^2 + 2x + 8$

(iv) $(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x+2}{4-5x}$

$= \dfrac{(3x+2)(4+5x)}{16-25x^2} = \dfrac{15x^2+22x+8}{16-25x^2}$

Prob: 2

$f(x) = 2x,\quad g(x) = x+4,\quad h(x) = 5-x^3$

find (i) $(f+g)(2)$; $(h-g)(2)$; $(f\cdot g)(2)$, $\left(\frac{h}{g}\right)(2)$

(i) $(f+g)(2) = f(2) + g(2)$

$= 4+6$

$= \underline{10}$

(ii) $(h-g)(2) = h(2) - g(2)$

$= 5-8 - 6$

$= -9$

(iii) $(f\cdot g)(2) = f(2)\cdot g(2)$

$= 4\times 6$

$= 24$

(iv) $\left(\frac{h}{g}\right)(2) = \frac{h(2)}{g(2)} = \frac{5-2^3}{2+4} = \frac{-3}{6} = \underline{-\frac{1}{2}}$

~~Problem: 3~~

~~$f(x) = 3x - x + 4$~~

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Composite function: Let A, B and C are three non empty sets. Let f be a function from A to B and g be a function from B to C.

i.e. $f: A \to B$ and $g: B \to C$

Then the function $g \circ f : A \to C$ defined as

$$g \circ f(x) = g(f(x)), \forall x \in A$$

is called the ~~function~~ composite function of f and g.

$$\begin{tikzcd} A \arrow[r, bend left, "f"] \arrow[rr, bend right, "g \circ f"'] & B \arrow[r, bend left, "g"] & C \end{tikzcd}$$

<u>Exp:-</u> Let $f : A \to B$ and $g : B \to C$

$A = \{a, b, c\}$, $B = \{1, 2, 3\}$, $C = \{x, y, z\}$

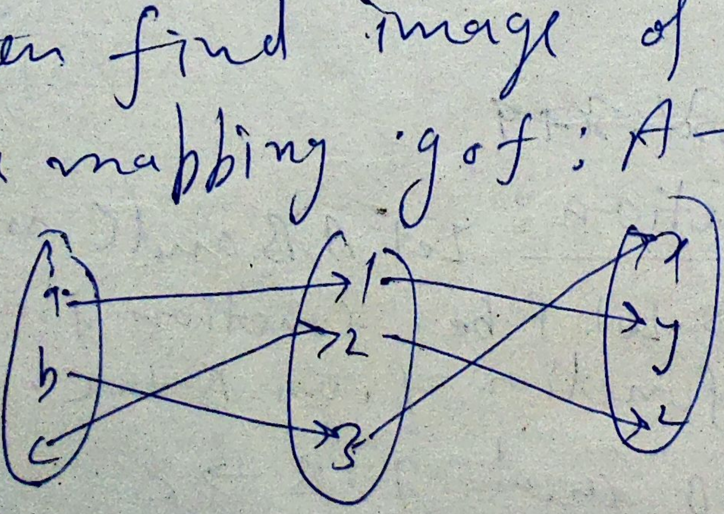
as

$f(a) = 1$ and $g(1) = y$

$f(b) = 3$ $\quad g(2) = z$

$f(c) = 2$. $\quad g(3) = x$

Then find image of a, b, c under the mapping $g \circ f : A \to C$.



$$g \circ f(a) = g(f(a))$$
$$= g(y)$$
$$= y$$

$$g \circ f(b) = g(f(b))$$
$$= g(3)$$
$$= x$$

$$g \circ f(c) = g(f(c))$$
$$= g(2)$$
$$= z$$

<u>Exp:-</u> Let $f: R \to R$ be defined by $f(x) = 2x+3$ & $g: R \to b$ be defined as $g(x) = x^2$ then find (i) $g \circ f(x)$ (ii) $f \circ g(x)$

Ans (i) $g \circ f(x) = g(f(x))$
$$= g(2x+3)$$
$$= (2x+3)^2$$
$$= 4x^2 + 9 + 12x$$

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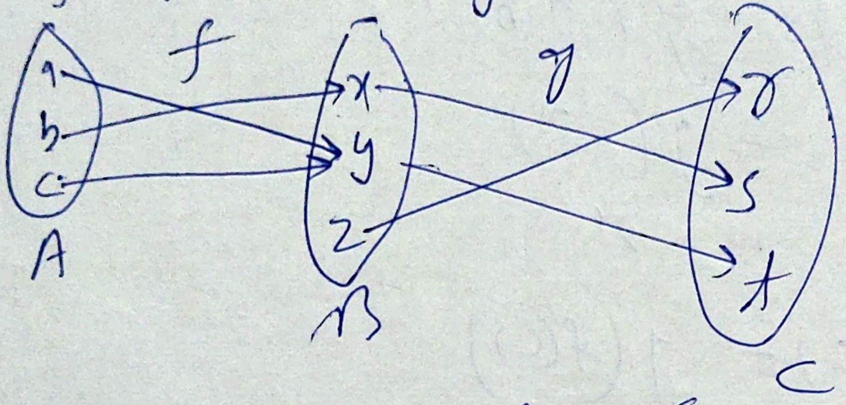
(ii) $f \circ g = f(g(x))$

$= f(x^2)$

$= 2x^2 + 3$

Exp:- $f: A \to B$ and $g: B \to C$



Find the $g \circ f: A \to C$

$g \circ f(a) = g(f(a))$

$= g(y)$

$= \cancel{t}$

$=$

$g \circ f(b) = g(f(b))$

$= g(x)$

$= s$

$g \circ f(c) = g(f(c))$

$= g(y)$

$= t$

Exp:-

$f(x) = 2x + 1$

$g(x) = x^2 - 2$

$g \circ f$

$= g(f(x))$

$= g(2x+1)$

$= (2x+1)^2 - 2$

$= 4x^2 + 1 + 4x - 2$

$= 4x^2 + 4x - 1$

an

Recursive function:-

~~factorial function:~~ (7)

① If $n = 0$ then $n! = 1$

② If $n > 0$, then $n! = n \times (n-1)!$

4!

fibonacci Sequence:-

(i) if $n=0$ or $n=1$ then $f(n) = n$

(ii) if $n > 1$ then $f(n) = f(n-1) + f(n-2)$

Ackermann function:-

① If $m = 0$ then $A(m,n) = n+1$

② if $m \neq 0$ but $n = 0$ then $A(m,n) = A(m-1, 1)$

③ if $m \neq 0$ and $n \neq 0$ then $A(m,n) = A(m-1, A(m, n-1))$

Find $\boxed{A(1,3) = 5}$

Q) $$Q(a,b) = \begin{cases} 0 & \text{if } a < b \\ Q(a-b, b) + 1 & \text{if } b \leq a \end{cases}$$

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Find $Q(2,5)$, $Q(12,5)$

0 2

Q) $$L(n) = \begin{cases} 0 & \text{if } n = 1 \\ L\lfloor n/2 \rfloor + 1 & \text{if } n > 1 \end{cases}$$ find $L\,25 = 4$

$A(1,4) = A(0, A(1,3))$

$A(1,3) = \cancel{A(0} \; A(0, A(1,2))$

$A(1,2) = A(0, A(1,1))$

$A(1,1) = A(0, A(1,0))$

$A(1,0) = A(0,1)$

$= 1$

$A(1,1) = A(0,1)$

$= 2$

$A(1,2) = A(0,2)$

$= 3$

$A(1,3) = A(0,3)$

$= 4$

$A(1,4) = A(0,4)$

$= 6$

Mathematical Induction: Unit:-1

Let S(n) be a Statement, we use three steps.

(i) Basic step - S(1) is true.

(ii) Hypothesis step - Let S(k) is true.

(iii) Induction step. S(k+1) is true for S(1) and S(k)

Exp:- ①

$P(n) = 1+3+5+ \cdots +(2n-1) = n^2$

L.H.S. $S(1) = 1$ R.H.S $S(1) = 1^2 = 1$ true

Let S(k) is true

$1+3+5+ \cdots +2k-1 = k^2$

Induction step.

S(k+1)

L.H.S. $\underbrace{1+3+5+ \cdots +2k-1}$ $+2(k+1)-1 = (k+1)^2$

L.H.S $= k^2 + 2(k+1) - 1$

$= (k+1)^2$

= R.H.S Prove

② $2+4+6+ \cdots 2n = n(n+1)$

$S(1) = 2$

R.H.S $= 1(1+1) = 2$ L.H.S = R.H.S

Let S(k) is true

$2+4+6+ \cdots 2k = k(k+1)$

S(k+1) $\underline{2+4+6+ \cdots +2k} + 2(k+1) = (k+1)(k+2)$

L.H.S $=$ ~~2+4~~ $k(k+1) + 2(k+1)$

$= (k+1)(k+2)$

= R.H.S.